

# Optimisation-based Identification of Situation Determined Cost Functions for the Implementation of a Human-like Driving Style in an Autonomous Car

Sven Kraus<sup>◇</sup>, Sebastian Albrecht<sup>\*</sup>, Marion Sobotka<sup>\*</sup>, Bernd Heißing<sup>◇</sup>, Michael Ulbrich<sup>\*</sup>  
Technische Universität München

<sup>◇</sup>Lehrstuhl für Fahrzeugtechnik, <sup>\*</sup>Lehrstuhl für Mathematische Optimierung  
Garching, Bavaria, D-85748, GERMANY  
Phone: +49-89-289 <sup>◇</sup>-15355, <sup>\*</sup>-17966  
Fax: +49-89-289 <sup>◇</sup>-15357, <sup>\*</sup>-17932  
E-mail: <sup>◇</sup>kraus@ftm.mw.tum.de, <sup>\*</sup>albrecht@ma.tum.de

The acceptance of (partly) autonomous driving systems depends critically on the obtained level of anthropomorphic behaviour, consequently the controller design has to be derived from the human archetype. Therefore a bilevel optimisation approach is applied to obtain the cost function humans minimise by their lateral driving style. The resulting optimal cost function for a given subject and the traffic situation is used to generate human-like control sequences for the autonomous car. An emotion model accounting for different situations in combination with a hierarchical guidance architecture is used to allow an online application of the results.

Topics/ Driver Behaviour & Driver Model, Vehicle Control, Driver-Vehicle System

## 1. INTRODUCTION

The vision of autonomous driving comes more and more into reach due to numerous research projects. These use experimental vehicles equipped with actuators and sensors to measure and accomplish movements and to show suitable driving behaviour under various conditions.

Although remarkable progress has been made in the field of autonomous driving (see e.g. DARPA Urban Challenge), passengers are not comfortable with current autonomous cars due to an unnatural driving style, i.e. a driving performance differing from the average human one. This inevitably leads to an effect called ‘Uncanny Valley’, which was primarily discovered in the 1970’s in the field of robotics. Today, it constitutes the phenomenon that the acceptance of the behaviour of technical systems depends on the degree of reality of the artificial systems. For the general acceptance of an advanced driver assistance system, especially for (partly) autonomous systems, the main goal is to design the situation adaptation as anthropomorphic as possible.

Therefore, this paper focuses on the question how to realise the preferably closest possible imitation of human-like driving behaviour in an autonomous car, especially in lateral dynamics. Utilising an emotion-based approach, the actually perceived traffic situation is classified and associated with parameters of the lateral vehicle dynamics controller (Section 2). This specific set of parameters corresponds to a combination of cost functions obtained by using a bilevel programming approach of nonlinear optimisation theory (Section 3). This allows a human-orientated mapping of situation to driving style. Finally, Section 4 shows a numerical example.

## 2. SITUATION ADAPTATION

To adapt to a new situation one has to change one’s manner of action. This aspect can be tackled by integrating emotions into the adaptation process. In this context, Ben-Ze’ev notes in [1]:

‘From a philosophical and a psychological point of view, emotions are not theoretical states; they involve practical concerns, associated with readiness to act. Since emotions are evaluative attitudes, involving a positive or a negative stance towards the object, they also entail either taking action or being disposed to act in a manner compatible with the evaluation.’

Furthermore, the following definition of the term ‘situation’ represents our requirements to an artificial interpretation system for traffic situations: The system has to manage a dynamic environment consisting of one or more actors that interact with each other with respect to certain rules.

One model that copes with these requirements and is also strongly geared to the emergence of human emotions triggered by particular aspects of the traffic situation, is the emotion model according to Ortony, Clore and Collins [2].

### 2.1 Emotion Model

The approach represents sensor information and variables deriving from conventional situation interpretation modules on an abstract level to emulate the cognitive structure of human emotions. Thereby, amount and complexity of sensor data can be reduced to a smaller number of inner states.

The approach takes object properties, agent behaviour and situation aspects in relation to normative and context-sensitive evaluation models into account. Such models are e.g. used in software-agents [3], humanoids [4], machine vision applications [5] and driver models in driving simulators [6]. So far, they have never been used as central assessment instance in a real world action-perception-loop to realise situation adaptive driving behaviour in an autonomous car by adapting the vehicle motion controllers.

### 2.2 Vehicle Guidance Architecture

It is intuitive to base the necessary vehicle guidance system and its architecture on the human archetype in order to develop a powerful autonomous vehicle guidance system replacing humans in the control loop to automate the entire driving mission. The model of Rasmussen [7] uses three different levels of cognitive behaviour corresponding to capabilities on different levels of abstraction. This capability-based and hierarchically organised architecture (according to Fig. 1) is implemented in our autonomous car.

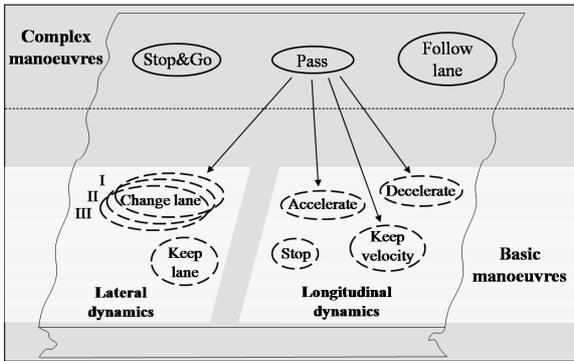


Fig. 1: Capability Network

This approach combines several advantages in the development of a vehicle guidance system and provides the basis for the execution of tasks commanded by decision-making instances [8]. The basic idea behind this concept is the decomposition of complex manoeuvres into simpler basic modules of driving (basic manoeuvres), that could be combined and parameterised according to the actual requirements.

### 2.3 Mapping Problem

The modular structure of the capability network offers the possibility to realise different characteristics of a capability node (cp. 'Change lane' type I-III in Fig. 1) by adapting the controller typology and/or parameters according to the actual requirements and circumstances. The chosen approach for interpretation of various situations allows to link the inner emotion state with a corresponding behaviour within the capability network (Fig. 2). Using this human-like process of situation adaptation, a more natural driving behaviour can be obtained on the lowest level of vehicle guidance.

Up until now no quantitative correlations between

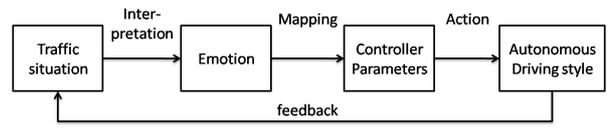


Fig. 2: Adaptation Process

emotional state and valid cost functions of the human lateral driving behaviour (considering the human as a control system) have been presented in literature.

In a first step, based on a safety-related view, the emotional states of a dangerous and a stress-free situation were assigned to a strict path-following and a comfort-orientated controller, respectively. Realisations of this approach show promising performances [9, 10] and offer potential beyond the realised application to motorway driving. Most challenging problems are the integration of the latest research results of cognitive behaviour into the mapping between emotion state and driving behaviour as well as the quantification of human parameters for situation interpretation and danger recognition.

### 3. PERSONALISED MAPPING

To cope with these challenges and to obtain a mapping which directly represents the individual driving style of a human driver, a new approach is introduced, which allows a personalisation of the autonomous driving style (Fig. 3).

By analysing the individual driving style of a human in manually driven situations, it is possible to identify the cost functions the driver minimises during these time intervals. With the help of dynamic programming, these cost functions lead to certain controller parameters. The designed controller thereby shows the same dynamic control behaviour as the before analysed driver within these situations. The definition of the latter is unique within this approach since the situation is defined by the states of the emotion model that was introduced in section 2.1. By correlating emotion states with controller parameters, it is then possible to link situations with individual human-like dynamic control behaviour. Thus, a personalisation of the autonomous driving style can be obtained.

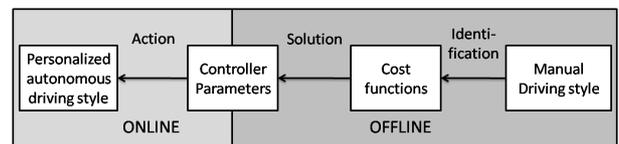


Fig. 3: Personalisation Process

Subsequently, we will not focus on the design of the individual controller sets and the correlation with emotion states, but on the analysis of the driving style. The goal is to find the cost function which an individual human driver minimises over a certain time interval taking the dynamics of the car into account. This mathematical problem can be stated as a so-called **bilevel program**

which is a combination of two optimisation problems, where one, the *lower level problem*, is part of the constraints of the *upper level problem*. Since 1973 many different approaches to solve these kinds of programs have been presented, see e.g. [11, 12].

After defining the model of the car and some basic cost functions humans might be minimising, we will state the bilevel program for our setting and present a solution strategy.

### 3.1 Car Model

The single-track car model utilised here is based on the model of Gerdtts et al. [13], where the equations of motion are given by the following system of ordinary differential equations (ODEs). The position of the car is given by  $(x, y)$ , its orientation by  $\psi$  and the steering angle is denoted by  $\delta$ .

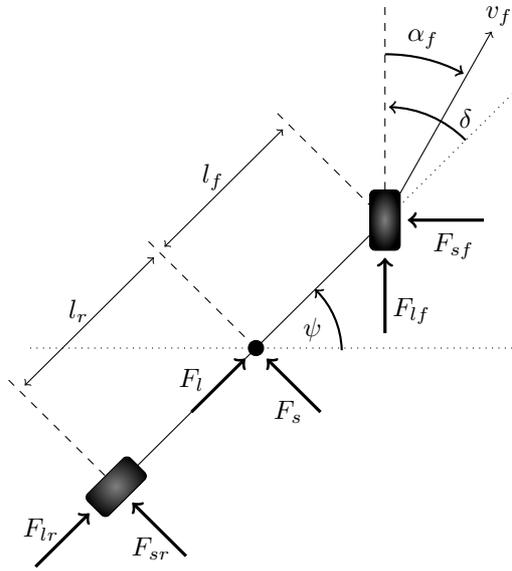


Fig. 4: Geometry of single-track car model

The equations of motion read:

$$\begin{aligned} x' &= v_x \\ y' &= v_y \\ \psi' &= \omega_\psi \\ v_x' &= \frac{1}{m} (F_l \cos(\psi) - F_s \sin(\psi)) \\ v_y' &= \frac{1}{m} (F_l \sin(\psi) + F_s \cos(\psi)) \\ \omega_\psi' &= \frac{1}{I_{zz}} (F_{sf} l_f \cos(\delta) - F_{sr} l_r + F_{lf} l_f \sin(\delta)) \\ \delta' &= \omega_\delta, \end{aligned}$$

where the forces  $F_l$  and  $F_s$  on the centre of mass are given by

$$\begin{aligned} F_l &= F_{lr} - F_A + F_{lf} \cos(\delta) - F_{sf} \sin(\delta), \\ F_s &= F_{sr} + F_{lf} \sin(\delta) + F_{sf} \cos(\delta). \end{aligned}$$

$F_A$  is the drag caused by air resistance,  $F_{lf}$  and  $F_{sf}$  are the longitudinal and lateral forces on the front tire, the respective forces on the rear tire are  $F_{lr}$  and  $F_{sr}$ . These forces depend on the state variables  $z := (x, y, \psi, v_x, v_y, \omega_\psi, \delta, \omega_\delta)^T$ , compare [13], and the acceleration (braking) force  $F_B$ . According to Pick and Cole [14] the arm and steering dynamics can be modelled by

$$c_2 \dot{\omega}_\delta + c_1 \omega_\delta + c_0 \delta = T_m - c_3 \alpha_f,$$

where  $\alpha_f$  is the front slip angle (cp. [13]), the control is the muscular torque  $T_m$  and constants are denoted by  $c_i$ .

Summing up, the combined car, arm and steering dynamics can be written as the following nonlinear system of ODEs:

$$z' = f(z, u),$$

with the control  $u = (F_B, T_m)^T$ .

Using a uniform time-discretisation, the dynamics are rewritten in the form  $h(\bar{z}, \bar{u}) = 0$ . To get a well-posed problem boundary conditions are needed in addition to the dynamics and the cost function. The considered boundary conditions  $z(t_0) = z_0$  and  $z(t_f) = z_f$  with given vectors  $z_0$  and  $z_f$  can be included in the equation  $h(\bar{z}, \bar{u}) = 0$ .

### 3.2 Basic Cost Functions

In the following some basic cost functions are stated which a human driver could minimise on a motorway ride. The focus of our numerical examples (Section 4) will be on lane-change manoeuvres, therefore the following cost functions are considered.

A family of basic cost functions is given by the integral of squared state variables (see Fig. 5), for example for the steering angle  $\delta$ :

$$f_\delta = \int_{t_0}^{t_f} \delta(t)^2 dt$$

Accordingly, if a uniform discretisation of time is used, the sum of the squared state variable is gained, for instance:

$$f_\delta = \Delta \sum_{i=1}^{N-1} \delta(t_i)^2,$$

where  $\Delta = t_{i+1} - t_i$  is the discretisation step width. For the considered example the following state variables were utilised: the velocity  $v_y$  in  $y$ -direction, the steering angle  $\delta$ , the yaw angle  $\psi$  and its time-derivative  $\omega_\psi$ .

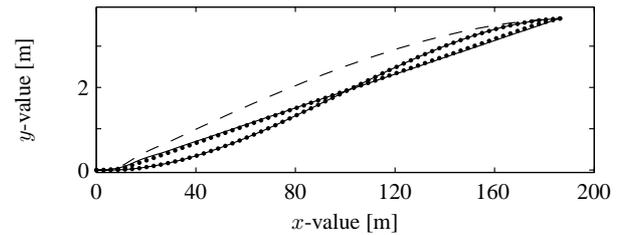


Fig. 5: Optimal trajectories for lower level costs:  $f_\psi$  (dotted),  $f_{v_y}$  (solid),  $f_{\omega_\psi}$  (dash-dotted) and  $f_\delta$  (dashed)

Sporrer et al. in [15] analysed human lane-change manoeuvres and found basic conditions, which can be linked to a family of basic trajectories. These can be described by piecewise quintic polynomial functions and three polynomials denoted by  $P_i$ , where  $i \in \{0.7, 0.5, 0.35\}$  states the concatenation point, are considered here (cp. Fig. 6). The lateral distance between the car position and the respective polynomial is used to gain a basic cost function  $f_{P_i}$ .

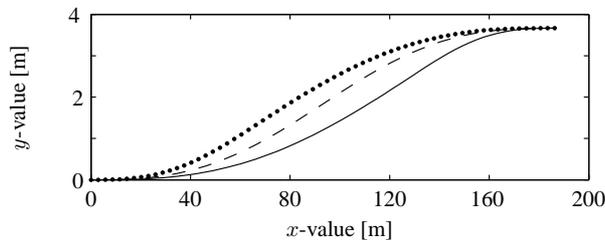


Fig. 6: Optimal trajectories for lower level cost:  $f_{P_{0.7}}$  (solid),  $f_{P_{0.5}}$  (dashed) and  $f_{P_{0.35}}$  (dotted)

### 3.3 Bilevel Problem

The basic assumption of our approach [16] is that humans minimise a cost function which can be approximated by a linear combination of the above mentioned basic costs  $f_i$ :

$$f = \sum_i w_i f_i,$$

utilising the weights  $w_i \in [0, 1]$  which have to fulfil the condition  $\sum_i w_i = 1$  to avoid ambiguity.

The lower level problem is to minimise the cost  $f$  for a given weight vector  $w$  subject to the constraints. Consequently, if  $f$  is the true human cost function, then the optimum of the lower level problem  $(\bar{z}, \bar{u})$  should reproduce the observed human actions. So in our case the lower level problem reads:

$$\min f(\bar{z}, \bar{u}|w) \text{ subject to } h(\bar{z}, \bar{u}) = 0.$$

Given recorded human driving data  $p_d$  the upper level problem is to find the weights  $w_i$  minimising the squared distance between  $p_d$  and  $p_c(\bar{z}, \bar{u}|w)$ , where  $\bar{z}$  and  $\bar{u}$  are the solutions of the lower level problem corresponding to the chosen weights:

$$\begin{aligned} \min F(\bar{z}, \bar{u}, w) &:= \text{dist}(p_d - p_c(\bar{z}, \bar{u}|w))^2 \\ &= \sum_{i=1}^N (p_d(t_i) - p_c(\bar{z}_i, \bar{u}_i|w))^2 \end{aligned}$$

with respect to:  $h(\bar{z}, \bar{u}) = 0$  holds and  $f(\bar{z}, \bar{u}|w)$  is minimised by  $\bar{z}$  and  $\bar{u}$ . In [16] the existence of a global optimum of a similar bilevel program is proven under non-restrictive assumptions.

### 3.4 Solution Strategy

To solve the bilevel problem the first-order necessary optimality conditions of constrained optimisation theory (KKT-conditions) are applied to the lower level problem:

$$\nabla f + \nabla h \lambda = 0$$

with  $\lambda$  being the Lagrange multipliers.

These conditions are now coupled to the upper level problem, transforming the bilevel program into a one-level problem:

$$\begin{aligned} \min F(\bar{z}, \bar{u}, w) &= \|p_{data} - p_{comp}(\bar{z}, \bar{u}|w)\| \\ \text{subject to} & \\ 0 &= \nabla f(\bar{z}, \bar{u}|w) + \nabla h(\bar{z}, \bar{u})\lambda \\ 0 &= h(\bar{z}, \bar{u}) \end{aligned}$$

Note that the bilevel program and the one-level problem are not equivalent, because the KKT-condition is only necessary but not sufficient for a non-convex problem.

This nonlinear optimisation problem is solved with the interior-point optimiser IPOPT [17]. For a similar setting it was shown in [16] that the one-level problem fulfils all prerequisites needed by the solver.

## 4. NUMERICAL EXAMPLE

An Audi Q7 equipped with several sensors (radar, lidar, cameras, dGPS and an inertial measurement unit) and actuators is used to perform autonomous driving tasks and to measure human driving behaviour. Fig. 7 shows some examples of recorded steering characteristics during lane change manoeuvres.

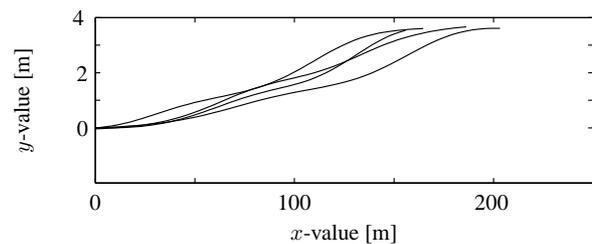


Fig. 7: Lane Change Steering Characteristics

### 4.1 Weight Scaling

The weighted sum of the basic cost functions yields the lower level cost  $f = \sum_i w_i f_i$  (cp. Section 3.3), but for the following interpretations of the optimisation results the weights  $w_i$  have to be rescaled due to the fact that the cost functions  $f_i$  are of different orders of magnitude.

Let  $\bar{z}^i$  and  $\bar{u}^i$  denote the optimal values of the lower level problem for the basic cost function  $f_i$ , then the following equation holds:

$$f_{il} := f_i(\bar{z}^i, \bar{u}^i) \leq f_i(\bar{z}^j, \bar{u}^j) \quad \forall j.$$

As a reference value the arithmetic mean

$$f_{im} := \frac{1}{n} \sum_{i=1}^n f_i(\bar{z}^i, \bar{u}^i)$$

is considered. Consequently, the weights  $\tilde{w}_i$  with respect to the scaled versions  $\tilde{f}_i$  with

$$\tilde{f}_i := \frac{1}{f_{im} - f_{il}} f_i$$

are of interest, since their values correspond to the observed combination of cost functions.

#### 4.2 Optimisation Results

To solve the bilevel problem with the optimiser IPOPT [17] initial values for  $\bar{z}$ ,  $\bar{u}$  and  $\bar{\lambda}$ , which, in combination with the vector of weights  $w_i$ , constitute the upper level state, are needed. The convergence of the program strongly depends on the choice of the initial values and since only local optima can be determined, a bad choice might lead to a by far suboptimal solution. Consequently, the solution of a lower level problem with a basic cost function was chosen which comes closest to the recorded human data. This approach guarantees that the starting values for  $\bar{z}$ ,  $\bar{u}$  and  $\bar{\lambda}$  fit to each other and so provides a starting point for the upper level problem with only small violation of the primary constraints.

In the following, two examples of lane-change manoeuvres on a motorway with speeds above 27.5 meters per second are discussed. Both times the car is driven by the same subject, but the situations differ considerably. The first example is recorded at a stressful moment, where the traffic situation only allows a fast lane-change, while the second one is a regularly paced manoeuvre in a comfortable traffic situation. Here the need to change the lane can be anticipated in advance and no other car has direct influence on the lane-change manoeuvre.

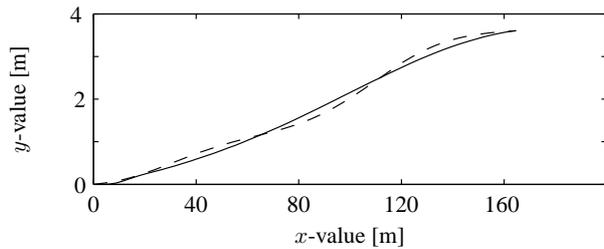


Fig. 8: Trajectory of car in example 1: recorded (dashed) and optimised (solid)

The above figure (Fig. 8) shows the recorded human-steered trajectory of the car (dashed line) and the optimal solution of the bilevel problem (solid line) for the first manoeuvre. The lateral distance of 3.6 metres is covered in 5.6 seconds, which is about 1 second less than the average value of 6.5 seconds. The car is driven 164.7 metres at a mean velocity of about 29.4 metres per second. Consequently, this is a fast lane-change in a stressful emotional state caused by other participants on the motorway.

The optimisation results show that a combination of  $f_\delta$  and  $f_{P_{0.7}}$  minimises the distance to the recorded data. The corresponding weights are  $\tilde{w}_\delta = 0.91$  and  $\tilde{w}_{P_{0.7}} = 0.09$ .

With 6.6 seconds the second manoeuvre has an average duration, at a mean velocity of 28.2 meters per second. The distances covered are 3.66 metres in  $y$ -direction and about 186.3 metres in  $x$ -direction. In Fig. 9 the recorded car positions are displayed by the dashed line and the corresponding trajectory obtained by bilevel optimisation is represented by the solid line.

The dominant cost functions are  $f_{\omega_\psi}$  and  $f_\delta$  with weights  $\tilde{w}_{\omega_\psi} = 0.32$  and  $\tilde{w}_\delta = 0.54$ , respectively. Besides these two only the comparatively small weights

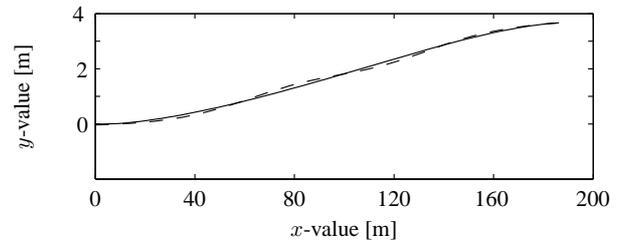


Fig. 9: Trajectory of car in example 2: recorded (dashed) and optimised (solid)

$\tilde{w}_{v_y} = 0.09$  and  $\tilde{w}_{P_{0.7}} = 0.05$  are non-zero.

Consequently, the optimisation results for the two examples differ considerably, as expected from the different scenarios of a fast and an average lane-change. The influences of  $f_{\omega_\psi}$  and  $f_{v_y}$  are comfort-oriented and lead to a smooth, continuous manoeuvre. On the other hand, the minimisation of  $f_{P_{0.7}}$  yields a lane-change with more changes of direction in the second part of the movement, which is far less comfort-oriented.

#### 5. FUTURE RESEARCH

In the future larger experimental studies will be done to show that for the same subject and a similar traffic situation the combinations of cost functions have only small variation. Additionally, a goal is to analyse the differences between subjects and to consider other driving manoeuvres beside lane-changes. Finally, if optimal combinations of basic cost functions are available for all situations and manoeuvres, the control strategies have to be tested on the real car.

#### 6. CONCLUSION

The realisation of autonomous-driven cars is a huge challenge from both the technical and the control perspective. To achieve sufficient acceptance by passengers and the required comprehension by other traffic participants new control strategies have to be applied. A key to generate acceptable controls is the situation adaption.

Therefore, we combined an emotion model accounting for different traffic situations with a hierarchical guidance architecture in our autonomous car. A bilevel problem was formulated to obtain the optimal combination of basic cost functions humans minimise by their driving behaviour. A solution strategy was presented and numerical examples showed that lane-changes in different traffic situation correspond with different combinations of cost functions.

As a result of the presented approach the driving behaviour of the autonomous car will be personalised and more anthropomorphic. Furthermore the situation aspects having significant influence on the control behaviour can be identified.

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**APPENDIX**

Figure 10 shows a picture of the Audi Q7 having been made available by Transregional Collaborative Research Center 28 *Cognitive Automobiles*.



Fig. 10: Experimental Vehicle Audi Q7

The final tables state some of the characteristic constants of this vehicle and the steering dynamics [14].

Property		Value
Inertia	$I_{zz}$	5600 [kg m <sup>2</sup> ]
Mass	$m$	3000 [kg]
Distance rear axis		
to center of gravity	$l_r$	1.5 [m]
Distance front axis		
to center of gravity	$l_f$	1.5 [m]
Front cornering stiffness	$c_f$	130000 [N/rad]
Rear cornering stiffness	$c_r$	255000 [N/rad]

Tab. 1: Constants used in single-track car model

Constant	Value
$c_0$	0.348
$c_1$	0.089
$c_2$	0.013
$c_3$	219.4

Tab. 2: Constants for the arm and steering dynamics (according to [14])