New spacetime discontinuous Galerkin methods for convection-diffusion systems

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We present new spacetime discontinuous Galerkin (DG) methods for solving systems of convection-diffusion equations, in particular the compressible Navier-Stokes equations. Our methods are extensions of an existing shock capturing streamline diffusion spacetime DG method [1] for solving hyperbolic conservation laws. The authors formulate the scheme in terms of entropy variables and use entropy stable fluxes, resulting in an entropy stable scheme.

In our extension to convection-diffusion equations we mostly follow the original scheme for the treatment of the non-linear convection terms. For the diffusion terms, we consider two approaches: the local discontinuous Galerkin (LDG) method [2] and the interior penalty (IP) method [3].
We examine theoretical properties of both schemes, such as the question whether the entropy stability of the original scheme transfers to the new schemes. We also compare the schemes numerically. Among other test problems, we show numerical results for the compressible Navier-Stokes equations in one space dimension [4].

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