Nonmonotone contact problems and their numerical simulation by using nonconvex Bundle method and regularization techniques

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Abstract

We consider problems with structual adhesive bonding which lead in their mathematical formulation to boundary value problems that involve nonmonotone and multivalued laws in the boundary conditions. These laws can be expressed by means of the Clarke subdifferential of a nonconvex, nonsmooth but locally Lipschitz function, the so-called superpotential. The variational formulation of these boundary value problems involving such laws gives rise to hemivariational inequalities with minimum superpotential. Since the mathematical analysis of the hemivariational inequalities is well established, we focus on their efficient numerical simulation. In general, there are two approaches for the numerical treatment of these nonconvex nonsmooth variational problems. The first one uses a regularization of the nonsmooth functional leading to a smooth variational problem, see [1]. The second one relies after discretization by finite element methods on nonsmooth optimization methods, like bundle types methods. We compare the nonconvex bundle method [3] with the regularization method [1, 2] in terms of their efficiency, and demonstrate the applicability of those methods in numerical simulation of the Double Cantilever Beam (DCB) Test Problems. We compute the unknown normal displacement on the contact boundary and the corresponding normal component of the stress vector. The latter is very important for the needs of the industry to predict the eventual failure of the structure by delamination.

References

[1] Ovcharova, N., *Regularization Methods and Finite Element Approximation of Hemivariational Inequalities with Applications to Nonmonotone Contact Problems,* PhD Thesis, Universit/"at der Bundeswehr Mnchen, Cuvillier Verlag, Göttingen 2012.

[2] Ovcharova, N., Gwinner. J., *A Study of Regularization Techniques of Nondifferentiable Optimization in View of Application to Hemivariational inequalities,* J. Optim. Theory Appl., DOI 10.1007/s10957-014-0521-y

[3] Dao, M.N., Gwinner, J., Noll, D. Ovcharova, N. *Nonconvex bundle method with application to a delamination problem,* submitted to SIAM J. Optimization