

Risk-Averse PDE-Constrained Optimization using Risk Measures

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In order to effectively model uncertainty or ambiguity in engineering and the natural sciences one typically considers partial differential equations (PDEs) with uncertain or distributed parameters. Passing from modeling and simulation to optimization, we arrive at stochastic optimization problems with PDE-constraints. This relatively new area requires a number of extensions from the classical stochastic optimization literature due to the infinite dimensional nature of the deterministic decision variables. This includes, e.g., well-posedness questions, derivation of optimality conditions, handling state constraints, and developing viable numerical methods. On the other hand, the uncertainty requires us to employ or extend concepts from risk management to assure risk-averse or robust decisions are met. In our work, we do this by using risk measures, e.g., conditional value-at-risk or mean plus upper-semideviation. Since these functionals are typically non-smooth, we suggest several smoothing techniques in order to make use of existing algorithms from PDE-constrained optimization. After motivating the model class, we discuss appropriate conditions on the objective functionals and derive first-order optimality conditions. We then demonstrate the effect of various risk-measures on the optimal solution via numerical experiments.