A multi-level adaptive solution strategy for 3D inverse problems in pool boiling

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Abstract

In this paper, an efficient algorithm based on a multi-level adaptive mesh refinement strategy is presented for the solution of ill-posed inverse heat conduction problems arising in pool boiling using few temperature observations. The stable solution of the inverse problem is obtained by applying the CGNE method together with a discrepancy stopping rule. The resulting three-dimensional PDE equations are solved numerically by a space-time finite element method. A multi-level computational approach, which uses an a posteriori error estimator to adaptively refine the meshes on different levels, is to speed up the entire inverse solution procedure employed. This systematic approach can efficiently solve the large-scale inverse problem considered without losing necessary detail in the estimated quantities. A simulation case study is set up to validate and assess the performance of the proposed solution approach with respect to computational efficiency and estimation quality. Moreover, effects of choosing different termination parameters for the iterative process at each mesh level are investigated. Finally, the algorithm is applied to the reconstruction of local boiling heat fluxes using real experimental data. The estimated surface heat flux offers a sound basis for the identification of dominant boiling heat transfer mechanisms and associated structured models.

Key words: Local heat flux estimation; Multi-level iterative regularization; Adaptive mesh refinement; Pool boiling; 3D IHCP

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### Nomenclature

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<tr>
<td>Ω</td>
<td>3D domain</td>
<td>[m$^3$]</td>
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<td>∂Ω</td>
<td>heater boundary</td>
<td>[m$^2$]</td>
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<tr>
<td>Γ$_H$</td>
<td>heated boundary</td>
<td>[m$^2$]</td>
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<td>Γ$_B$</td>
<td>boiling boundary</td>
<td>[m$^2$]</td>
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<td>Γ$_A$</td>
<td>adiabatic boundary</td>
<td>[m$^2$]</td>
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<td>outer normal on ∂Ω</td>
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<td>measurement data</td>
<td>[K]</td>
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<td>T$_m$</td>
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<td>q</td>
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<td>T$_f$</td>
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<td>ρ</td>
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<td>λ</td>
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<td>Δt$_s$</td>
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### 1 Introduction

Despite the enormous research effort in the past decades, the mechanistic understanding of boiling phenomena is still not yet well developed. Existing design methods are based on correlations which are only valid for one of the boiling regimes [1–4]. On the macroscale, a unifying framework for nucleate and transition boiling based on a macroscopic geometry model – the so-called “vapor stems” has been suggested in [5]. On the mesoscale, single bubbles growing on a heated plate or emerging out of the closed film in film boiling have been studied in detail [6–8]. On the microscale, the microlayer theory proposed in [9] predicts that most of the heat during boiling is transferred in the micro-region of the three-phase contact line by evaporation. Experimental research works to support this theory have been recently reported [10,11].

Among all kinds of studies on boiling processes, the estimation of the unmeasurable local heat fluxes at the boiling surface is considered an important prerequisite for the
accurate modeling and analysis of the boiling heat transfer mechanisms over the entire range of boiling conditions. Boiling heat flux has been considered to be correlated with various parameters, e.g. superheat, nucleate site density or bubble diameter in nucleate boiling and average vapor fraction or vapor velocity in transition boiling [1]. It is yet unclear which parameters dominate boiling heat transfer. A reliable prediction of local boiling heat fluxes is still not available, especially for higher heat flux boiling regimes. In order to precisely understand the fundamental local processes in pool boiling, high-resolution measurement techniques and corresponding efficient data processing methods are indispensable. In recent years, a powerful set-up for pool boiling experiments has been developed and operated by our collaboration partners at Technical University of Berlin in the group of H. Auracher. In their experiments, transient temperature evolutions have been observed at high temporal and spatial resolution by means of an array of micro-thermocouples (MTC) mounted below the solid-liquid contact surface [12]. These temperature measurements can be used to deduce the local surface boiling heat flux if a suitable computational method is available [13]. In combination with optical probe measurements [14], which can be used to identify the interfacial geometry of the two-phase flow [15], a database is available for the development of realistic mechanistic heat transfer models for boiling regimes beyond low heat flux nucleate boiling, where heat transfer models can be based on the behavior of a collection of single undisturbed bubbles. Moreover, local heat flux estimates may be directly used for a validation of boiling models on the microscale level [9].

In contrast to the reconstruction of the macroscopic boiling curve, which can be determined experimentally [16] or estimated computationally for example by means of the Coupled Map Lattice Method [17], this work focuses on the estimation and investigation of local boiling heat flux at each operating point along the entire boiling curve from high-resolution experimental data. The reconstruction of the heat flux distribution at the solid-liquid interface from measured temperatures inside a heater constitutes an inverse heat conduction problem (IHCP) [18,19]. The solution of this ill-posed problem [20] is generally not stable for small changes in the given data or even not unique if only finitely many measurements are available. In order to cope with this inherent ill-posedness, solution strategies typically rely on so-called regularization methods [21,22]. Due to the high complexity, most of the numerical and application-oriented IHCP papers are restricted to one or two space dimensions. For a detailed discussion of IHCP solution techniques refer to [18,19]. The few recent works on IHCP in three dimensions [23–26] are briefly reviewed as follows. A numerical algorithm for solving a 3D unsteady nonlinear IHCP defined in a
cylindrical geometry by means of an iterative regularization method is addressed in [23]. In [24], a spherically symmetric 3D IHCP is solved using a modified Tikhonov regularization method. In [25], a new procedure involving dynamic state observers is reported to solve multidimensional IHCP. Furthermore, specialized methods for the solution of anisotropic 3D IHCP have been recently reported in literature [26] and applied to investigate how the heat transfer in falling films is influenced by the wave characteristics. An efficient iterative regularization strategy for a 3D IHCP arising in pool boiling has been introduced recently by the authors [27]. This method is not limited to special computational domains, e.g. cylinder, sphere or thin plate, but generally applicable to different types of heat conductor geometries employed in practical applications. By means of a suitable choice of the regularization term, the solution method can cope with both idealized (high-resolution) and realistic measurement data which are always restricted in resolution. Furthermore, this method is very efficient, especially for large-scale IHCP, which are computationally intractable for many other solution techniques. In the solution approach [27], the conjugate gradient method for the normal equation (CGNE) together with the discrepancy stopping rule is used to obtain a regularized stable heat flux estimate. The non-uniqueness of the solution, resulting from a limited number of spatial point observations, is resolved by considering a so-called $H^1$ regularization technique [27]. Moreover, the use of space-time finite element methods allows the fast numerical solution of the resulting direct, adjoint and sensitivity problems to facilitate the treatment of the entire heater in three space dimensions. However, the method is restricted to a fixed uniform discretization of the spatial domain. An adaptation of the spatial grid gives room for further improvement. Such an adaptive method is the focus of this work.

Adaptive mesh refinement strategies can in principle reduce the total computational effort for the direct and inverse problems significantly without losing the desired estimation quality. The use of adaptive algorithms based on a posteriori error estimation has been reviewed in the context of finite element discretization of partial differential equations (PDE) in [28] for example. Such strategies have recently attracted increasing interest in the optimization and inverse problems community. In [29,30], an error estimator based on an optimal control approach to a posteriori error estimation [31,32] has been reported for the solution of optimization problems constrained by a stationary PDE. This error estimation technique has been extended to optimization problems governed by a system of parabolic PDE [33]. More recently [34], an efficient inexact Newton algorithm has been studied for determining an optimal regularization parameter in Tikhonov regularization according to the discrepancy principle. In particular, an adaptive discretization strategy based on
the concept of goal-oriented error estimators is introduced for parameter identification in PDE problems. Furthermore, a complete convergence analysis has been carried out in the context of Tikhonov regularization. In these papers, only 2D stationary or transient problems have been treated by adaptive discretization. We will build on these works and extend the method to general 3D IHCP problems which have to be typically solved for the interpretation of experimental data in boiling research.

The boiling heat flux is non-uniformly distributed on the surface due to the strong local activity of the boiling process. Therefore, the use of a uniformly discretized mesh for inverse boiling problems generally leads to very high computational cost. An algorithm which can automatically refine the mesh locally during the solution procedure is hence very important for achieving the necessary estimation quality with small numerical effort. The contribution of this work is the development of a multi-level adaptive mesh refinement strategy to further improve the computational efficiency of a method we have reported previously [27]. The proposed solution approach decomposes the entire iterative solution procedure into several sub-procedures, in which a sequence of gradually refined meshes are used. An easy-to-implement a posteriori error estimator is evaluated to indicate how to locally refine a given mesh. By means of this approach, much less iterations are required for the computation at finer mesh levels. Hence, the entire iterative procedure terminates in a much faster manner. Large-scale 3D transient IHCP in pool boiling at long observation times, previously computationally intractable, can be efficiently solved for the first time by means of the suggested multi-level adaptive discretization method. A simulation case study and the solution of the inverse boiling problem using real experimental data show that for a given estimation quality much less computational effort is required by the adaptive method compared to a method using only a single fine mesh. To our knowledge, the boiling heat transfer is determined by phenomena on a much smaller scale than nowadays resolved by available measurement techniques. The multi-level adaptive mesh refinement strategy proposed in this work can cope with highly resolved local detail at low computational effort. It thus constitutes an enabling technology even for problems of larger size resulting from the application of higher resolution measurement techniques, which are expected to become available in future boiling research.

The paper is organized as follows. In Section 2, the mathematical formulation of the IHCP arising in pool boiling and its solution strategy on the continuous level are given. A multi-level computational approach is presented in Section 3. An adaptive mesh refinement strategy using an a posteriori error estimation technique is addressed next in Section 4. In the first part of Section 5, the performance of this solution approach is validated by
a simulation case study. The importance of choosing suitable termination parameters for
different mesh levels is demonstrated in its second part. Finally, the proposed solution
approach is applied to real measurement data obtained during pool boiling experiments
at TU Berlin in the group of H. Auracher. Conclusions and an outlook on our future work
are given in Section 6.

2 Mathematical formulation and solution strategy

In this section, the mathematical formulation of the IHCP in the context of pool boiling is
presented. The experiment in [12] is considered as a prototype. A brief description of the
experimental setup is given next. The core of the heater (cf. Fig. 1 (left)) is made of high
purity copper. It consists of a cylindrical part (35 mm diameter, 5 mm height) at the top,
and a flat part (38 mm × 38 mm, 2 mm high) with cut-off edges at the bottom. High-
resolution transient temperature measurements are taken by a 6 × 6 MTC array mounted
on a 1 mm × 1 mm square area located 3.6 µm below the boiling surface Γ_B in its center.
In addition, a few more MTC are distributed in the vicinity of the MTC array also at a
distance of 3.6 µm below the boiling surface Γ_B (cf. Fig. 1 (right)). The top layer of the
heater of total thickness of 3.6 µm consists of a copper layer of 2.5 µm thickness sputtered
on the top surface. To avoid oxidation and corrosion, the boiling surface is coated with a
gold layer of 1 µm thickness, which is separated from the copper core by a titanium layer
of 0.1 µm thickness acting as a diffusion resistance layer. For further details refer to [12].

As shown in Fig. 1 (right), transient temperature observations Θ_m := Θ|_M are mea-
sured at N_m = 44 point locations x ∈ M ⊂ Ω, where Ω denotes the 3D domain of the
heater geometry. In practice, only perturbed data are available. In the following, these
noisy temperature observations are denoted by Θ^δ_m with δ being an upper bound on the
measurement noise. Since the temperature measurements obtained in the experiment are
highly resolved in time but are only available at a few finite spatial locations, the function
space \( Y = L^2(0, t_f; \mathbb{R}^{N_m}) = \{Y_i(t) ∈ L^2(0, t_f) : i = 1, \ldots, N_m\} \) is chosen as the data space.
To enforce uniqueness of the inverse problem solution, the unknown heat flux \( q \) is sought
in the function space \( X = L^2(0, t_f; H^1(Γ_B)) \) (cf. [27] for details).

The inverse problem under investigation corresponds to the reconstruction of the unknown
heat flux \( q \) from temperature measurements \( Θ^δ_m \). It is formulated as follows:

\[
\text{Find } q ∈ X \text{ such that } Aq = Θ^δ_m, \quad (1)
\]
Fig. 1. Left: The entire heater geometry. Right: The x-y positions of 44 numbered temperature sensors, which are located 3.6 $\mu$m below the boiling surface $\Gamma_B$.

where the operator $A$ is implicitly given by

$$\rho c_p \frac{\partial \Theta}{\partial t} = \nabla \cdot (\lambda \nabla \Theta), \quad \text{in } \Omega \times (0, t_f), \quad (2)$$

$$\Theta(\cdot, 0) = \Theta_0(\cdot), \quad \text{on } \Omega, \quad (3)$$

$$\lambda \frac{\partial \Theta}{\partial n} = q_h, \quad \text{on } \Gamma_H \times (0, t_f), \quad (4)$$

$$\lambda \frac{\partial \Theta}{\partial n} = 0, \quad \text{on } \Gamma_A \times (0, t_f), \quad (5)$$

$$\lambda \frac{\partial \Theta}{\partial n} = q, \quad \text{on } \Gamma_B \times (0, t_f). \quad (6)$$

The boundary of the heater is defined as $\partial \Omega = \Gamma_H \cup \Gamma_B \cup \Gamma_A$ (cf. Fig. 1 (left)). The observation time interval is $[0, t_f]$. $\rho$, $c_p$ and $\lambda$ denote density, heat capacity and heat conductivity of the material, respectively. Since the temperature variation in pool boiling experiments is only within a few Kelvin, it suffices to model these material properties as temperature-independent quantities. The initial condition and the boundary conditions at $\Gamma_i$, $i \in \{H, A\}$, are assumed to be known. Due to the affine-linear nature of the equation system (2)–(6), the inverse problem can be reduced as follows:

Find $q \in \mathcal{X}$ such that $Kq = T_m^h$, \quad (7)

where the operator $K$ is linear and implicitly given by

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T), \quad \text{in } \Omega \times (0, t_f), \quad (8)$$

$$T(\cdot, 0) = 0, \quad \text{on } \Omega, \quad (9)$$

$$\lambda \frac{\partial T}{\partial n} = 0, \quad \text{on } (\Gamma_H \cup \Gamma_A) \times (0, t_f), \quad (10)$$
\[ \lambda \frac{\partial T}{\partial n} = q, \]  
on \Gamma_B \times (0, t_f). \quad \text{(11)}

\( T := \Theta - T_d \) depends linearly on the unknown \( q \). Further, \( T^\delta_m := \Theta^\delta_m - T_d|_M \) with temperature \( T_d \) resulting from a solution of eqs. (2)–(6) with \( q = 0 \). Since \( T_d \) does not depend on the unknown quantity \( q \), it can be precomputed before the inverse solution procedure starts. Now, only the reduced linear inverse problem (7) needs to be solved to reconstruct the unknown heat flux \( q \) at the boiling surface \( \Gamma_B \). Note that the corresponding measurement data \( T^\delta_m \) and its noise-free counterpart satisfy the relation

\[ \|T_m - T^\delta_m\|_Y \leq \delta. \quad \text{(12)} \]

To summarize, the IHCP stated in eq. (7) can be cast into the following minimization problem:

\[ \min J(q) \quad \text{(13)} \]

subject to

\[ J(q) = \|Kq - T^\delta_m\|^2_Y > (\tau \delta)^2. \quad \text{(14)} \]

\( Kq \) corresponds to the temperature predictions at the measurement locations for a given heat flux \( q \) which results from a solution of the model (8)–(11). The constraint (14) is necessary to avoid an overfitting. It bounds the residual from below by the measurement error. \( \tau > 1 \) is a parameter and its choice is to be decided (cf. Section 4). The norm in eqs. (12) and (14) is defined by

\[ \|Y\|^2_Y := \frac{1}{N_m} \sum_{i=1}^{N_m} \frac{1}{t_f} \int_0^{t_f} Y_i(t)^2 \, dt. \quad \text{(15)} \]

The solution of the minimization problem (13)–(15) is presented below in detail.

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3 Multi-level iterative regularization

The CGNE algorithm [21], an iterative method, is in some sense optimal for the solution of the minimization problem (13)–(15). This algorithm has been used in our previous work on the solution of IHCP in boiling research [27]. The details of this algorithm are not repeated here. In the context of this algorithm, the constraint (14) can be interpreted as a suitable stopping rule to decide on an appropriate termination of the descent iteration of the minimization algorithm. It implements the well-known discrepancy principle [21] which suggests to stop the iterative process when the difference between the computed and
measured temperatures matches roughly the error magnitude (cf. eq. (12)). The iteration is terminated at iteration $k^*$ if the prediction error for the current approximate solution $q_{k^*}$ is getting close to the measurement error, i.e. if

$$\|Kq_{k^*} - T_m^\delta\|_Y \leq \tau \delta.$$  

For the numerical solution of the (continuous) IHCP (13)–(15), the PDE model [13,27] has to be discretized. Straightforwardly, these equations can be replaced by discrete approximations on a sufficiently fine mesh in order to satisfy a certain degree of accuracy. A space-time finite element discretization [27,35] is used for this purpose. The discrete version of the objective functional (13) is given by

$$J(\hat{q}) := \|\hat{K}\hat{q} - T_m^\delta\|_Y^2,$$

where $\hat{q}$ denotes the projection of $q$ onto the finite-dimensional space defined by the finite element discretization and $\hat{K}$ stands for the corresponding operator acting on $\hat{q}$.

Given the strong local variations frequently occurring in boiling heat transfer, the choice of the discretization mesh requires some consideration. The use of a very fine fixed mesh resolving the local variation would result in a very high numerical effort. Moreover, it is difficult to know whether a mesh is sufficiently fine or not before the computation is started since no a priori knowledge about the characteristics of the boiling heat flux is available in general. Therefore, it is important to develop algorithms which can automatically construct non-uniform meshes during the computational process. Here, a multi-level adaptive mesh refinement strategy in conjunction with a limited number of point-wise temperature measurements inside the test heater is proposed. A very simple strategy is favored in this paper to minimize the additional computational effort of the adaptation. The main idea of the suggested approach is to decompose the entire iterative procedure into several sub-procedures, in which the meshes on different levels are gradually refined only locally close to the measurement positions to decrease the discretization error in the temperature solution. The proposed multi-level scheme is summarized in Algorithm 1.

The inverse solution procedure is started on a coarse mesh $\mathcal{T}_h^1$ with an initial guess of $\hat{q}_0^1 = 0$ (step 1) due to a lack of better information. In step 2, a stable solution $\hat{q}^l$ on mesh $\mathcal{T}_h^l$ is computed by applying the CGNE-based iterative regularization approach [27]. An error estimator (to be discussed in the next section) is used in step 3 as a local refinement indicator from one level to the next. If the pre-defined stopping condition is fulfilled
(step 4), the algorithm is terminated or alternatively the meshes $\mathcal{T}_h^l \rightarrow \mathcal{T}_h^{l+1}$ are refined and the present approximate solution is used as an initial guess for the solution on the next finer mesh $\mathcal{T}_h^{l+1}$ (step 5). The sequence of steps 2-5 are repeated (step 6) until the stopping condition is fulfilled. Algorithm 1 shows that once an initial mesh is selected, the subsequent finer meshes will be automatically constructed during an adaptation loop. The rationale of this strategy is as follows: Although the estimated heat flux on a coarser mesh level is a yet unacceptable approximation, it is a good initial guess for the computations on the next mesh level. Consequently, fewer iterations should be required for the computations on subsequent finer meshes. Since only local refinement is performed during the computation, the meshes constructed on different levels generally result in much fewer grid points than a pre-defined fine uniform mesh. The total computational effort can hence be significantly reduced.

Algorithm 1: Multi-level adaptive regularization

1. Choose an initial coarse mesh $\mathcal{T}_h^1$ and set $l = 1$, $\hat{q}_0^1 = 0$.
2. Solve the inverse problem on $\mathcal{T}_h^l$ to obtain a stable solution $\hat{q}^l$.
3. Evaluate an $a$ posteriori error estimator.
4. If a predefined stopping condition is fulfilled, stop.
5. Refine $\mathcal{T}_h^l \rightarrow \mathcal{T}_h^{l+1}$ and set the initial value $\hat{q}_0^{l+1} = \tilde{q}^l$, with $\tilde{q}^l$ being the interpolation of the solution $\hat{q}^l$ on the finer mesh $\mathcal{T}_h^{l+1}$.
6. Increment $l$ and go to 2.

In the following section, the implementation of step 3 in the multi-level adaptive algorithm is presented in detail, since this is the main algorithmic contribution of this paper.

4 $a$ posteriori error estimation

During recent years, $a$ posteriori error estimation techniques have been proposed by several authors [29,30,34,36] in the context of inverse problems. In [29,30], a general concept for an $a$ posteriori estimation of the discretization error with respect to an arbitrary functional is presented and applied to stationary optimal control problems in the context of combustion. The integration of the error estimation technique into a solution method for inverse problems relying on Tikhonov regularization is addressed in [34] with an application to the solution of a 2D elliptic boundary value problem. In [36], the gradient of the least-squares objective functional is used as a mesh refinement and coarsening indicator.
for the estimation of a distributed transmissivity parameter in a 2D parabolic PDE.

Here, an alternative and in a sense pragmatic adaptive local mesh refinement strategy is presented to solve general 3D IHCP with limited measurement information which arise in pool boiling. It is based on an error estimator to control the error between the computed and the measured temperatures at given measurement points. This approach is straightforward and easy to implement. Since only solutions of the forward problem at the measurement locations are required for an evaluation of the error estimator, there is no extra computational cost. Moreover, it can easily be integrated with the CGNE-based iterative regularization method proposed in our previous work [27]. The error estimator at mesh level \( l \) considers the temperature residual at each spatial measurement location \( i \),

\[
\eta_l^i := \| \hat{K} \hat{q}^l|_i - T_m^\delta|_i \|, \quad i = 1 \ldots N_m,
\]

with the norm definition

\[
\| \cdot \|_2 = \frac{1}{t_f} \int_0^{t_f} (\cdot)^2 \, dt.
\]

\( T_m^\delta|_i \) and \( \hat{K} \hat{q}^l|_i \) correspond to the measured and the predicted temperatures for a given stable solution at spatial location \( i \) at computational level \( l \), respectively. Applying the triangle inequality and using eq. (16) gives

\[
\eta_l^i = \| \hat{K} \hat{q}^l|_i - Kq^l|_i \| + \| Kq^l|_i - T_m^\delta|_i \|
\leq \| \hat{K} \hat{q}^l|_i - Kq^l|_i \| + \| Kq^l|_i - T_m^\delta|_i \|
\leq \eta_h^l + \tau \delta.
\]

\( Kq^l|_i \) corresponds to the continuous perfect solution at location \( i \) and \( \eta_h^l \) denotes the upper bound of discretization errors at all measurement locations at computational level \( l \). Hence, the residual comprises a first contribution resulting from discretization (\( \eta_h^l \)) and a second due to measurement noise (\( \tau \delta \)). While the level of measurement error \( \delta \) can be determined experimentally prior to the boiling experiment, an estimate of the discretization error is only possible during the computations, since it depends on the actual non-uniform mesh and type of discretization. If the computation of an error estimate is to be avoided to limit the computational effort, the upper bound in eq. (19) can be replaced by \( C_l \delta \) which implicitly defines the quantity \( C_l \) depending on both, the measurement and the discretization error:

\[
C_l = \tau + \frac{1}{\delta} \eta_h^l.
\]

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The iterative process at level $l$ is then terminated if
\[ \eta^l_i \leq C_l \delta \]  \hspace{1cm} (21)
with a constant $C_l$ to be provided by the user. Since the discretization error is a positive quantity $C_l > \tau > 1$ has to hold and the choice of $C_l$ depends on the mesh $T^l_h$.

With this error estimator, the multi-level adaptive algorithm is given as follows. The algorithm starts with a coarse initial mesh, which could result in a large discretization error dominating the total error, i.e. $\eta^l_h \gg \tau \delta$. Hence, a relatively large value for $C_l$ should be used at the initial mesh $l = 1$ to obtain a stable heat flux estimate. On the subsequent computational levels, the volume close to each measurement location will be refined or not depending on the computed error information. More explicitly speaking, if the temperature error at measurement location $i$ computed from (18) is larger than $C_l \delta$ after the inverse computation at level $l$, the volume close to sensor location $i$ will be locally refined such that the discretization error can be reduced on the next level. In contrast, if the temperature error is sufficiently close to $C_l \delta$, no refinement will be carried out around $i$. The discretization error will become smaller as meshes are chosen increasingly finer and the value of $C_l$ should be chosen smaller towards $\tau$ correspondingly. A detailed discussion on the selection of numerical values of $C_l$ for different mesh levels $l$ will be investigated in Section 5.2 in the context of an application.

Different from previous work [29,30,34,36], explicit \emph{a posteriori} error estimation on the quantity to be estimated, namely on the heat flux $q$ is not considered here. Instead, an error estimator dealing only with information on the temperature residuals is used. It is straightforward and easy to implement due to the measurement configuration considered. However, the proposed error estimator (18) does not provide an exact error bound for the heat flux solution $q$. Still, by considering the term reflecting the measurement noise in (19) at each computational level, a stable heat flux solution can be obtained. As will be shown in the following section, this adaptive solution approach results in good estimation results despite its conceptual shortcomings. The algorithm can automatically construct discretization meshes of suitable resolution to satisfy the necessary accuracy requirement. Compared to the use of a fine uniform mesh, the total computational effort can be significantly reduced.
5 Numerical results

In this section, the proposed multi-level adaptive mesh refinement strategy is first validated and assessed by a simulation case study. Then, the effect of the choice of the termination parameters $C_l$ for different mesh levels $l$ is analyzed. Finally, the adaptive mesh refinement strategy is applied to estimate the heat flux profiles from real temperature data taken in pool boiling experiments. All numerical computations are conducted on a computer with a 2.0 GHz AMD Athlon 64 processor. The finite element code NETGEN/NGSolve [37] is used to generate the finite element meshes and to compute the solutions of the arising sensitivity and adjoint PDE problems. The CGNE minimization algorithm combined with the proposed multi-level adaptive mesh refinement strategy is coded in C++ and integrated with the NETGEN/NGSolve software package.

5.1 Simulation case study

In order to assess the performance of adaptive mesh refinement, the real experiment is simulated accounting for the heater geometry (cf. Fig. 1 (left)), the thermal properties of the materials and the actual measurement locations [12]. For the numerical solution, the following values for density, thermal capacity, and thermal conductivity are used [38]: $\rho^C = 8933$ kg m$^{-3}$, $c^C_p = 397$ J kg$^{-1}$ K$^{-1}$, and $\lambda^C = 393$ W m$^{-1}$ K$^{-1}$ for copper, and $\rho^G = 19,300$ kg m$^{-3}$, $c^G_p = 131$ J kg$^{-1}$ K$^{-1}$, and $\lambda^G = 311$ W m$^{-1}$ K$^{-1}$ for gold. These values will also be applied in the estimation procedure using real experimental data in Section 5.3. The simulation time horizon is chosen to be $t_f = 30$ ms and an equidistant time step of $\Delta t_s = 0.04$ ms is selected. The noise-free temperature measurements are generated by solving eqs. (8)–(11) with $q = q_{ex}$ numerically on a sufficiently fine mesh with 113,367 vertices and 470,260 elements (cf. Fig. 2 (left)) using polynomials of order 1. A heat-flux test-function

$$q_{ex}(t, x, y) = a(t) \cdot b(x, y),$$

is chosen to simulate three heat-flux peaks at the center of the boiling surface above the MTC locations. The functions $a(t)$ and $b(x, y)$ are defined as

$$a(t) = 8 \cdot \sin(\pi t/30),$$

$\ldots$
and

\[ b(x, y) = \sum_{i=1}^{N_b=3} e^{-(a_i(x-x^c_i)^2+b_i(x-x^c_i)(y-y^c_i)+c_i(y-y^c_i)^2)}, \]

where \( \{a_i, b_i, c_i\} \) are parameters to control the orientation and shape of three Gaussian peaks whose centers are located at \((x^c_i, y^c_i)\). Each peak represents a rising bubble on the top of the test heater. In this case study, these parameters are chosen explicitly as \( \{a_1 = 10.25, b_1 = 4.5, c_1 = 10.25, x^c_1 = 0.4887, y^c_1 = -0.0620\} \), \( \{a_2 = 8, b_2 = 0, c_2 = 8, x^c_2 = 0.4255, y^c_2 = 2.068\} \) and \( \{a_3 = 10.25, b_3 = -4.5, c_3 = 10.25, x^c_3 = -1.1, y^c_3 = -2\} \), respectively. The functions \( a(t) \) and \( b(x, y) \) are illustrated in Fig. 3 for later reference. The peak values of the simulated heat flux are 8 MW m\(^{-2}\). The noisy measurement data are constructed by perturbing the exact values with an artificial measurement error, which is generated by multiplying a given standard deviation \( \delta \) with a zero mean normal
Fig. 4. Left: Contour plot of the simulated heat flux at $t = 15$ ms. Right: Contour plot of the estimated heat flux at $t = 15$ ms using a fine mesh $\mathcal{T}_h^{\text{fine}}$ as shown in Fig. 2 (right). ‘+’ represent the measurement locations on the boundary of MTC 1–36 and at MTC 37–44 below the surface (cf. Fig. 1 (right)).

The distribution of variance one. $\delta$ is chosen here as 0.025 K, which reflects the noise level apparent in the experimental data (cf. Section 5.3).

Table 1
A comparison of numerical results using a single-level computational strategy and the multi-level adaptive mesh refinement strategy.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>elements</th>
<th>$C$</th>
<th>$J \times 10^{-3}$</th>
<th>iterations</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{T}_h^{\text{fine}}$</td>
<td>140,071</td>
<td>1.02</td>
<td>0.63</td>
<td>33</td>
<td>4625.5 s</td>
</tr>
<tr>
<td>$\mathcal{T}_h^1$</td>
<td>3156</td>
<td>4</td>
<td>9.84</td>
<td>20</td>
<td>31.8 s</td>
</tr>
<tr>
<td>$\mathcal{T}_h^2$</td>
<td>9340</td>
<td>2</td>
<td>2.15</td>
<td>18</td>
<td>811 s</td>
</tr>
<tr>
<td>$\mathcal{T}_h^3$</td>
<td>11,458</td>
<td>$\sqrt{2}$</td>
<td>1.20</td>
<td>12</td>
<td>67.5 s</td>
</tr>
</tbody>
</table>

In this case study, three computational levels are considered. The initial mesh $\mathcal{T}_h^1$ (cf. Fig. 5 (a)) is chosen to reflect the measurement resolution. Two subsequently refined meshes $\mathcal{T}_h^2$, $\mathcal{T}_h^3$ (cf. Fig. 5 (b),(c)) are automatically constructed by the adaptation strategy using the error information computed at the previous levels. The termination parameters $C_l$ for the sequences of meshes are chosen decreasingly as $C_1 = 4$, $C_2 = 2$ and $C_3 = \sqrt{2}$, respectively. An explanation of this choice will be given in Section 5.2. For demonstrating the efficiency of the multi-level adaptive approach, the considered IHCP is also solved on a single fine mesh $\mathcal{T}_h^{\text{fine}}$ with 33,446 vertices and 140,071 elements (cf. Fig. 2 (right)) for comparison. Numerical results comparing the computational efficiency and the estimation
Fig. 5. (a)–(c) Contour plots of the estimated heat fluxes in units [MW m\(^{-2}\)] at \(t = 15\) ms obtained at different levels \(T_h^l, l = 1, 2, 3\).
Table 2  
Quantitative study of heat flux reconstructions in the area above the dense $6 \times 6$ MTC array.

<table>
<thead>
<tr>
<th></th>
<th>Max [MW m$^{-2}$]</th>
<th>Min [MW m$^{-2}$]</th>
<th>Mean [MW m$^{-2}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>8</td>
<td>0</td>
<td>0.71</td>
</tr>
<tr>
<td>single-level</td>
<td>7.72</td>
<td>-0.58</td>
<td>0.70</td>
</tr>
<tr>
<td>multi-level</td>
<td>8.38</td>
<td>-0.90</td>
<td>0.73</td>
</tr>
</tbody>
</table>

quality are given in Tab. 1 as well as in Fig. 4 (right) and in Fig. 5, respectively.

The estimated heat flux using the single-level strategy employing a fine mesh (cf. Fig. 4 (right)) captures the major dynamics of the simulated heat flux well, especially in the area above the $6 \times 6$ MTC array. As expected, the inverse algorithm yields smoothed heat flux distributions with broader peak shapes and smaller peak values. The errors are much higher in the vicinity of MTC 38 and MTC 43 than those in the vicinity of MTC 1–36 due to the much lower spatial resolution.

The estimated heat flux using the multi-level strategy is shown in Fig. 5. The heat flux solution obtained at the first level (cf. Fig. 5 (a)) has a broader middle high heat flux area and its peak values on other two high heat flux areas are higher compared to those of the estimation results using the single fine mesh $T_{h}^{\text{fine}}$ (cf. Fig. 4 (right)). Nevertheless, this estimated heat flux at the first level is already a good initial guess for the next computational level. After two refinements, the optimal estimated heat flux is finally obtained at the 12th iteration on mesh $T_{h}^{3}$. Because of the good initial guess obtained from the first mesh level, fewer iterations are required for the inverse computation on subsequent finer mesh levels. The total computational time (180.4 s) using the three-level adaptive strategy is significantly reduced compared to that (4625.5 s) using the single fine mesh $T_{h}^{\text{fine}}$ only, while their estimation results are comparable with respect to both the peak values and the contour of the peaks in the high heat flux areas. (cf. Fig. 4 (right) and Fig. 5 (c)). In Tab. 2, the exact heat fluxes with the reconstructions using different computational strategies are compared. For assessing the reliability of the reconstructions, extremal and mean heat fluxes are given. The reconstructions of the total respectively mean fluxes are comparable for both variants. In both cases (using the single-level and multi-level strategies), some small negative values occur in the estimates. They are due to numerical artefacts resulting from the ill-posed nature of the inverse problem considered.
5.2 Choice of termination parameters $C_l$

In this subsection, the effect of different $C_l$ values in the stopping criterion (21) based on the discrepancy principle at each mesh level is investigated. Starting from eq. (20) one can show by simple manipulations that

$$C_{l-1}/C_l = (\tau + \frac{1}{\delta} \eta_{h-1})/(\tau + \frac{1}{\delta} \eta_h) = \left(\frac{\delta \tau}{\eta_h} + \frac{\eta_{h-1}}{\eta_h}\right)/\left(\frac{\delta \tau}{\eta_h} + 1\right). \tag{22}$$

Obviously, the sequence of termination parameters $C_l$ can be determined through (20) and (22) if the discretization error at each computational level were known. However, the explicit evaluation of the discretization error $\eta_h$ is difficult and computationally expensive if unstructured meshes are employed. It is interesting to note that the sequence of termination parameters is determined by the sequence of discretization errors only if the measurement error $\delta$ is negligible. The $C_l$ values are chosen heuristically in this work. Numerical experiments are presented below to illustrate the effect of the choice on the estimation quality.

Fig. 6 shows that the inverse solution obtained at the initial mesh level becomes temporally fluctuating as $C_1$ approaches 1. It is shown in Fig. 7 (multi 1) that the fluctuation can
Fig. 7. Temporal evolution of estimated heat fluxes at location \((x, y) = (0.4887, -0.0629)\) obtained using the three-level adaptive computational strategy with different values of \(C_l\); "multi 1": \(\{C_1 = C_2 = C_3 = 1.02\}\), "multi 2": \(\{C_1 = C_2 = C_3 = 4\}\), "multi 3": \(\{C_1 = C_2 = C_3 = 10\}\), "multi 4": \(\{C_1 = C_2 = C_3 = 20\}\), "multi 5": \(\{C_1 = 4, C_2 = 2, C_3 = \sqrt{2}\}\).

not be removed from the inverse solutions obtained at subsequent computational levels. Hence, it is not appropriate to choose values of \(C_1\) close to 1 at the initial mesh. On the other hand, choosing large values of \(C_l\) for all levels will lead to an early stopping of the iterative process to result in underestimated results (cf. Fig. 7 (multi 2–4)). Based on the above analysis, the use of relatively large values for \(C_1\) at the initial level and then a decreasing sequence of values at the subsequent levels is suggested. The rationale behind this choice is that the discretization error is smaller after the adaptive mesh refinement from one level to the next. This indicates that the corresponding values of \(C_l\) should be chosen to converge to 1 as \(l\) increases in order to avoid underestimation effects. Note that this heuristic strategy can also yield good results even if \(C_1\) is assigned with larger values such as 10 or 20, although more computational effort is then required at subsequent mesh levels due to the underestimated initial guess obtained at the initial level.

The following heuristic is suggested to construct a decreasing sequence of values for \(C_l\):

\[
C_l = \sqrt{C_{l-1}}, \quad l = 2, 3, \ldots, \tag{23}
\]

where \(l\) corresponds to the mesh level. Other choices, e.g. a linearly or exponentially
Fig. 8. Boiling curve of isopropanol at $P_{sat} = 0.1$ MPa. The 9 operating points used in this study are highlighted.

decreasing sequence is also applicable. For comparison, the alternative strategies using the sequences \{\text{C}_1 = 4, \text{C}_2 = 2, \text{C}_3 = \sqrt{2}\}, \{\text{C}_1 = 4, \text{C}_2 = \frac{12}{5}, \text{C}_3 = \frac{36}{25}\} and \{\text{C}_1 = 4, \text{C}_2 = 4e^{-0.5}, \text{C}_3 = 4e^{-1}\} have been tested for the multi-level computation. All of them yield estimation results of similar quality. Note that the sequence \{\text{C}_1 = 4, \text{C}_2 = 2, \text{C}_3 = \sqrt{2}\} has been used in the simulation case study in Section 5.1. It will also be employed for the multi-level estimation procedure applied to real data in the following subsection.

5.3 Heat flux estimation using measurements from pool boiling experiments

In this subsection, measurement data sets obtained during a pool boiling experiment with isopropanol [12] are considered. The computational domain, the material properties and the measurement locations have already been introduced in Section 5.1. The temperature observations using MTC have been recorded at a sampling frequency of 25 kHz. Hence, an equidistant step size of $\Delta t_s = 0.04$ ms is used for time discretization. The total observation time for all measurement data sets is chosen to be 100 ms, which amounts to 2500 time steps. In our previous work, the observation time was restricted to 30 ms due to the computational bottleneck of the solution approaches proposed in [13,27]. Since there are no experimental data for the initial temperature available and the values of measured temperatures at the first time instant are almost the same at the measurement locations, their average value is employed as an estimate of the initial temperature of the test heater. It has been shown in [13] that such an estimate has negligible influence on the overall
Table 3
Numerical results for processing experimental data obtained at the operating points 1–9 using the multi-level adaptive mesh refinement strategy for \( C_1 = 4 \), \( C_2 = 2 \) and \( C_3 = \sqrt{2} \).

<table>
<thead>
<tr>
<th>Nr.</th>
<th>mesh level</th>
<th>elements</th>
<th>( J \times 10^{-3} )</th>
<th>iterations</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>10.0</td>
<td>7</td>
<td>35.7 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>9468</td>
<td>2.41</td>
<td>20</td>
<td>272.9 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>12,103</td>
<td>1.21</td>
<td>17</td>
<td>301.9 s</td>
</tr>
<tr>
<td>2</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.53</td>
<td>15</td>
<td>73.9 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>9630</td>
<td>2.47</td>
<td>42</td>
<td>602.4 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>11,779</td>
<td>1.20</td>
<td>36</td>
<td>630.6 s</td>
</tr>
<tr>
<td>3</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.96</td>
<td>38</td>
<td>177.6 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>8133</td>
<td>2.49</td>
<td>67</td>
<td>759.8 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>11,898</td>
<td>1.21</td>
<td>37</td>
<td>648.8 s</td>
</tr>
<tr>
<td>4</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.80</td>
<td>76</td>
<td>353.3 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>8066</td>
<td>2.47</td>
<td>76</td>
<td>841.4 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>12,612</td>
<td>1.24</td>
<td>39</td>
<td>735.0 s</td>
</tr>
<tr>
<td>5</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.92</td>
<td>47</td>
<td>223.7 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>8954</td>
<td>2.48</td>
<td>68</td>
<td>845.8 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>12,186</td>
<td>1.21</td>
<td>37</td>
<td>664.2 s</td>
</tr>
<tr>
<td>6</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.65</td>
<td>25</td>
<td>120.2 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>9352</td>
<td>2.47</td>
<td>53</td>
<td>702.7 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>11,450</td>
<td>1.23</td>
<td>38</td>
<td>636.3 s</td>
</tr>
<tr>
<td>7</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.68</td>
<td>13</td>
<td>64.4 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>9873</td>
<td>2.50</td>
<td>42</td>
<td>596.6 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>12,734</td>
<td>1.22</td>
<td>34</td>
<td>638.9 s</td>
</tr>
<tr>
<td>8</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>8.03</td>
<td>3</td>
<td>17.8 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>7643</td>
<td>2.43</td>
<td>11</td>
<td>125.8 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>11,354</td>
<td>1.17</td>
<td>13</td>
<td>225.6 s</td>
</tr>
<tr>
<td>9</td>
<td>( \mathcal{T}_h^1 )</td>
<td>3156</td>
<td>9.87</td>
<td>6</td>
<td>31.8 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^2 )</td>
<td>9791</td>
<td>2.44</td>
<td>18</td>
<td>258.0 s</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{T}_h^3 )</td>
<td>12,848</td>
<td>1.20</td>
<td>17</td>
<td>336.9 s</td>
</tr>
</tbody>
</table>
Fig. 9. Temperature data $T^\delta_m$ used for solution procedure of the reduced inverse problem (cf. Section 2) at MTC 17 at the 9 different operating points as shown in Fig. 8.

estimation quality. A constant heat flux $q_h$ is considered on $\Gamma_H$. This is due to the fact that the heat supply measured in the experiment is almost stationary and uniformly distributed in space. The proposed multi-level adaptive computational strategy is used to process the temperature data recorded at 9 operating points along the entire boiling curve of isopropanol (cf. Fig. 8). The noise level is chosen as $\delta = 0.025$ K which is based on the analysis of estimation results obtained in [13] by means of an iterative regularization via a L-curve stopping criterion [39].

The numerical statistics of all computations are given in Tab. 3. These data show the efficiency of the proposed adaptation method. The value of the objective function is decreased significantly from one level to the next, which is due to the increasing resolution of the refined grid. In all cases, the total computational effort is (well) below 2000 s. Hence,
Fig. 10. Estimated boiling heat fluxes above MTC 17 at the 9 different operating points as shown in Fig. 8.

the evaluation of the experimental data can easily be integrated into the experimental work process. This way information can be gained to guide the selection of the improved experimental conditions during measurement computation. Note that a solution on the fine mesh shown in Fig. 2 (right) fails due to memory limitations on the computer used.

Fig. 9 displays the temporal evolutions of $T_{m}^{\delta}$ at MTC 17 to provide an example of the measurement data used for the solution of the IHCP at nine different operating points (1)–(9) along the boiling curve of isopropanol (cf. Fig. 8). Fig. 10 shows the corresponding estimated boiling heat fluxes above MTC 17. Furthermore, estimated surface boiling heat fluxes (MW m$^{-2}$) are shown in Fig. 11 for different operating points and in Fig. 12 for operating point 3 at selected time instants. While Fig. 11 provides insight into the different patterns of the local heat fluxes in different boiling regimes, Fig. 12 illustrates
the evolution of the estimated boiling heat fluxes obtained at operating point 3, close to critical heat flux, during a short period of time. In the following, the estimation results in different boiling regimes along the boiling curve shown in Fig. 8 are discussed.

In very low heat flux nucleate boiling, e.g. at operating point 1, the temperature fluctuations are moderate (cf. Fig. 9 (1)). The surface temperature exhibits fluctuations between 0.1 and 0.3 K, sometimes even less. The peak value of the estimated boiling heat flux (cf. Fig. 10 (1)) is expectedly also very small.

At higher heat fluxes in nucleate boiling towards the critical heat flux, e.g. at operating points 2 and 3, both the number of temperature fluctuations per unit time and their
Fig. 12. Contour plots of the estimated surface boiling heat flux at operation point 3 within the time interval [71.36 ms, 71.68 ms].

Amplitude increase. Distinct temperature fluctuations induced by the boiling process can be clearly distinguished from the measurement noise. Frequent sharp temperature drops with amplitudes up to 1.0-1.5 K are observed. The estimated corresponding heat fluxes (cf. Fig. 10 (2, 3)) have very high peak values of up to 10 MW m$^{-2}$. Moreover, small regions with high heat flux can be observed around a few MTC locations in Fig. 11 (2, 3). The temperature and heat flux fluctuations at these operating points are probably caused by evaporating liquid-vapor structures such as nucleating bubbles with dimensions smaller than the size of the MTC array. Since the temperature drops recorded at these operating points are similar to those observed in a single bubble nucleate boiling experiment [10], it might be possible that these temperature drops are caused by similar mechanisms as proposed in the microlayer theory [9]. In the related work [40], peak heat fluxes estimated from high-resolution temperature field measurements [10] are thirty times higher than the
macroscopic transferred heat flux through the boiling surface.

After passing the critical heat flux, the number of temperature fluctuations per unit time decreases in transition boiling (e.g. at operating points 4, 5 and 6). However, larger temperature drops are observed. From the estimation results shown in Fig. 11 (4, 5, 6), it is observed that the size of the high heat flux regions is gradually enlarged. This indicates that the temperature and heat flux fluctuations become more correlated above the MTC array as the average temperature superheat of the boiling surface is increased. In contrast to nucleate boiling, the sharp temperature drops and the resulting heat flux patterns in transition boiling are probably due to a rewetting front passing the boiling surface.

As the average temperature superheat further increases (e.g. at operating points 7, 8 and 9), the high heat flux areas are more extended (cf. Fig. 11 (7, 8, 9)), whereas their peak values decrease (cf. Fig. 10 (7, 8, 9)). The boiling process is going to enter the film boiling regime, in which the heat transfer rate decreases due to the increased presence of the vapour film.

In summary, the temporal and spatial evolution of the estimated local boiling heat flux across the boiling surface differs significantly among the boiling regimes. The time intervals with high temperature excursion continue to grow from high heat flux nucleate to transition boiling. Since time and space scales are coupled by the velocity of the wetting and rewetting process, longer periods of the temperature excursions mean larger structures of a possibly non-wetting area on the MTC array, due to the presence of a local vapour cluster for instance. Generally, in high heat flux nucleate boiling at lower superheats, the amplitudes of the heat flux fluctuations are smaller than those observed in transition boiling. However, the number density of such temperature drops is much larger in nucleate than in transition boiling. The magnitude and the number density of temperature and heat flux fluctuations is therefore a feature of a particular boiling regime. In nucleate boiling the temperature drops and the associated high heat fluxes are certainly caused by rapid local evaporation at a nucleation site on the heater surface, whereas they are most likely caused by liquid rewetting of highly superheated and vapour-covered surface spots in transition boiling. The characteristics of temperature and heat flux fluctuations in the different boiling regimes are associated with the dynamics of the two-phase layer above the heater surface. The corresponding measurements obtained by optical microprobes [14] yield information about vapor-liquid structures and to some extent interface velocities of the two-phase structure above the surface. Optical probe measurements can be used to identify the interfacial geometry of the two-phase flow and to correlate the
geometry of the interface with the boiling heat flux [15].

As mentioned in the introduction section, the identification of local boiling heat flux is indispensable for the complete modeling of boiling heat transfer. The peak heat fluxes estimated in this study in different boiling regimes reaches up to 10 MW m$^{-2}$ and exceed the average boiling heat flux by an order of $10^2$. Heat fluxes of such an order of magnitude have also been predicted in [9] for low heat flux nucleate boiling. This indicates the necessity of developing efficient computational methods which can reliably identify the local detail of the heat flux fluctuations. Since the proposed multi-level adaptive solution approach is very efficient, one can easily process the data sets while the boiling experiments are carried out during the investigation of the entire boiling curve. Many more data sets available from [12] have been processed. Their estimation results are not shown here because of space limitations.

6 Conclusions

In this work, a multi-level adaptive local mesh refinement strategy combined with a CGNE-based iterative regularization method is presented for the efficient solution of large-scale 3D IHCP arising in pool boiling. An easy-to-implement a posteriori error estimator has been considered as a local mesh refinement indicator. Its high computational efficiency has been illustrated by a simulation case study. Moreover, it has been shown that the choice of different values of $C_l$ in the discrepancy stopping conditions for each level is crucial for obtaining a good overall estimation quality. The proposed algorithm has also been applied to real experimental data in pool boiling. The high efficiency of this approach allows the fast processing of experimental data at many operating conditions along the entire boiling curve, which has been considered previously as computationally intractable.

The present study is our first step towards a systematic approach to consider an adaptive mesh refinement for the solution of large-scale inverse boiling problems. The integration of the presented temperature-based error estimation technique with a heat flux-based one will be investigated in future work, in order to assess the benefits of a more general a posteriori error estimation techniques for pool boiling IHCP. The obtained results will be compared to those of the present work with respect to computational efficiency and estimation quality.
It is well known, that the heat transfer is not only determined by the properties of the heater. Rather, the interaction with the multi-phase flow close to the heater surface is very relevant. The direct simulation requires the numerical simulation of multi-phase flow problems using multi-phase Navier-Stokes equations. For example, it has been shown in a direct simulation of spray cooling [41] that the major heat transfer is due to transient heat conduction. This indicates that the reconstructed transient local boiling heat flux distribution can provide useful information for further development of suitable models of boiling heat transfer. Its high computational efficiency via adaptive mesh refinement strategy establishes an important methodological step towards a heat transfer model which includes not only the heater but also the two-phase boundary layer, which we consider as a long-term goal.

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References


