ANALYSIS OF AN ELLIPTIC CONTROL PROBLEM WITH NON-DIFFERENTIABLE COST FUNCTIONAL

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We consider an optimal control problem of the type

$$
(\mathbf{P}) \qquad \begin{cases} \text{Minimize} & J(u) = \dfrac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \dfrac{\nu}{2}\,\|u\|^2_{L^2(\Omega)} + \mu\,\|u\|_{L^1(\Omega)} \\ \text{such that} & Ay + a(\cdot, y(\cdot)) = u \quad \text{in } \Omega \\ & y = 0 \quad \text{on } \partial\Omega \\ \text{and} & u_a \le u \le u_b. \end{cases}
$$

Here, $\Omega \subset \mathbb{R}^n$ is a bounded domain with $C^{1,1}$ boundary. The PDE is semilinear such that it admits a twice continuously differentiable control-to-state mapping. The other parameters satisfy

- $u_a \in (-\infty, 0)$ and $u_b \in (0, \infty)$,
- ν, $\mu > 0$,
- $y_d \in L^2(\Omega)$.

We point out that the problem ($\mathbf{P}$) is non-differentiable due to the L^1-norm and non-convex due to the nonlinear control-to-state mapping.

In this talk we derive first and second order optimality conditions for ($\mathbf{P}$). Due to the second order sufficient condition, the discretization error can be estimated. Numerical examples which confirm these estimates are presented.